Discrete Mathematics

UNIT-3

Permutations and Combinations:

Permutations and combinations are fundamental concepts in combinatorial mathematics. They deal with counting and arranging objects in different ways.

Permutations:

- A permutation is an arrangement of objects in a specific order.
- The number of permutations of a set of n objects taken r at a time is denoted by P(n, r) and is calculated as P(n, r) = n! / (n - r)!.
- For example, if you have 5 books and want to arrange 3 of them on a shelf, the number of permutations is P(5, 3) = 5! / (5 - 3)! = 60.

Combinations:

- A combination is a selection of objects without considering their order.
- The number of combinations of a set of n objects taken r at a time is denoted by C(n, r) and is calculated as C(n, r) = n! / (r! * (n - r)!).
- For example, if you have 5 books and want to select 3 of them to read, the number of combinations is C(5, 3) = 5! / (3! * (5 - 3)!) = 10.

Example of Permutations:

- Arranging the letters "ABC" in different orders: ABC, ACB, BAC, BCA, CAB, CBA.
- Finding the number of ways to arrange 4 people in a line: P(4, 4) = 4! = 24.

Example of Combinations:

- Selecting 2 fruits from a basket containing apples, oranges, and bananas: {apple, orange}, {apple, banana}, {orange, banana}.
- Choosing a committee of 3 members from a group of 7 people: C(7, 3) = 7! / (3! * (7 - 3)!) = 35.

Pigeonhole Principle and its Applications:

The pigeonhole principle, also known as the drawer principle, states that if you distribute more objects into fewer containers, then at least one container must contain more than one object. This principle finds applications in various areas of mathematics and computer science.

1. Example of the Pigeonhole Principle:
    - If you have 6 pigeons and only 5 pigeonholes, then at least one pigeonhole must contain more than one pigeon.

2. Applications:
    - Birthday Problem: In a group of 23 people, the probability that at least two of them have the same birthday is greater than 50%, which is a consequence of the pigeonhole principle.
    - Dirichlet's Drawer Principle: If you have 10 pairs of socks of different colors randomly placed in a drawer, then by choosing 11 socks, you are guaranteed to have at least one pair of socks of the same color.
    - Graph Coloring: In graph theory, the pigeonhole principle is used to prove the existence of a coloring for a graph such that no two adjacent vertices have the same color.

Types of Graphs:

Graph theory deals with the study of graphs, which consist of nodes (vertices) connected by edges. Here are some common types of graphs:

1. Undirected Graph:
    - An undirected graph is a graph in which edges have no orientation.
    - Example: A graph representing friendships between people, where the edges do not have a direction.
2. Directed Graph (Digraph):
    - A directed graph is a graph in which edges have a specific direction.
    - Example: A graph representing a road network, where the edges indicate one-way streets.
3. Weighted Graph:
    - A weighted graph is a graph in which edges are assigned weights or values.
    - Example: A graph representing distances between cities, where the weights on edges represent the distances.

4. Complete Graph:
    - A complete graph is a graph in which every pair of distinct vertices is connected by an edge.
    - Example: A graph with n vertices, where each vertex is connected to every other vertex.
5. Bipartite Graph:
    - A bipartite graph is a graph whose vertices can be divided into two disjoint sets such that no two vertices within the same set are adjacent.
    - Example: A graph representing the relationship between students and courses, where students are in one set and courses are in another set.

Walks, Paths, and Circuits:

In graph theory, walks, paths, and circuits describe different types of traversals within a graph.

1. Walk:
   - A walk is a sequence of vertices and edges in a graph, where each edge is adjacent to the previous and next vertices.
   - Example: A-B-C-D is a walk in a graph with vertices A, B, C, and D.
2. Path:
   - A path is a walk in which all vertices and edges are distinct.
   - Example: A-B-C-D is a path, but A-B-C-D-C is not a path since vertex C is repeated.
3. Circuit:
   - A circuit is a closed walk in which the first and last vertices are the same, and all other vertices and edges are distinct.
   - Example: A-B-C-D-A is a circuit in a graph.

Eulerian and Hamiltonian Graphs:

Eulerian and Hamiltonian graphs are special types of graphs with specific traversal properties.

1. Eulerian Graph:
   - An Eulerian graph is a graph that contains an Eulerian circuit, which is a circuit that visits every edge exactly once.
   - Example: A graph where each vertex has an even degree is Eulerian.
2. Hamiltonian Graph:
   - A Hamiltonian graph is a graph that contains a Hamiltonian cycle, which is a cycle that visits every vertex exactly once.
   - Example: A complete graph with more than two vertices is always Hamiltonian.

Shortest Path Algorithms:

Shortest path algorithms are used to find the most efficient path between two vertices in a graph. The most well-known algorithm for this purpose is Dijkstra's algorithm.

1. Dijkstra's Algorithm:
    - Dijkstra's algorithm finds the shortest path between a starting vertex and all other vertices in a weighted graph.
    - It uses a greedy approach, iteratively selecting the vertex with the minimum distance and updating the distances of adjacent vertices.
    - Example: Finding the shortest path between two cities in a road network, considering the distances between cities as edge weights.

Isomorphism of Graphs:

Isomorphism is a concept in graph theory that determines whether two graphs are structurally the same, even if their vertex and edge labels differ.

1. Graph Isomorphism:
    - Two graphs G and H are said to be isomorphic if there exists a bijection between their vertex sets, preserving adjacency.
    - Example: The graphs with vertices A-B-C and 1-2-3 are isomorphic if the edges between A, B, C correspond to the edges between 1, 2, 3.

Planar Graph:

A planar graph is a graph that can be drawn on a plane without any edges crossing.

1. Planar Graphs:

- Planar graphs are widely studied in graph theory and have many applications.
- Example: A tree is always planar since it has no cycles or closed loops.

2. Non-Planar Graphs:
    - Non-planar graphs are graphs that cannot be drawn on a plane without edge crossings.
    - Example: A complete graph with five or more vertices is non-planar.

These are some of the key concepts in discrete mathematics related to permutations and combinations, the pigeonhole principle, types of graphs, walks, paths, circuits, Eulerian and Hamiltonian graphs, shortest path algorithms, isomorphism of graphs, and planar graphs.

Questions

Permutations and Combinations:

1. In how many ways can a committee of 4 members be chosen from a group of 10 people?
2. How many 3-digit numbers can be formed using the digits 1, 2, 3, 4, 5 without repetition?
3. A team of 5 basketball players needs to be selected from a pool of 10 players. How many different teams can be formed?

Pigeonhole Principle and its Applications:

4. In a room, there are 8 people and 5 hats. According to the pigeonhole principle, at least how many people must wear the same type of hat?
5. A box contains 25 red balls and 20 blue balls. If you randomly select 46 balls, how many balls must be of the same color, according to the pigeonhole principle?

Types of Graphs:

6. How many edges does a complete bipartite graph with 5 vertices in each set have?
7. In a weighted graph with 6 vertices, how many different edge weights are possible if each edge weight is a positive integer less than or equal to 10?

Walks, Paths, and Circuits:

8. Consider a graph with vertices A, B, C, D, and E, and edges AB, BC, CD, DE, and EA. Find a path from vertex A to vertex E that visits each vertex exactly once.
9. In a graph with vertices A, B, C, D, E, and F, and edges AB, BC, CD, DE, and EF, find a circuit starting and ending at vertex A that visits each vertex exactly once.

Eulerian and Hamiltonian Graphs:

10. Determine whether the following graph is Eulerian or Hamiltonian: Vertices A, B, C, D, and E, and edges AB, BC, CD, DE, and EA.